INDIVIDUAL OPPORTUNITY THESIS

Recursive Asterisms

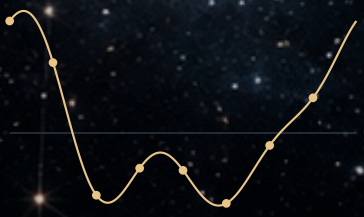
Explore the frontiers of mathematics.
Build on verifiable foundations.

$$F^*F = I$$

A reversible change of basis

01 · A pattern

$$f(\theta)$$



Eight values of one pattern

02 · A basis of rotations

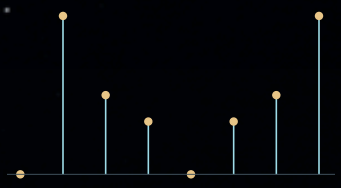
$$e^{i\theta}$$



Roots of unity

03 · Another representation

$$X = Fx$$



Magnitude and phase

$f(\theta) = \cos\theta + \frac{1}{2}\cos 2\theta + \frac{1}{3}\sin 3\theta;\ \theta = 2\pi j/8$

An invertible Fourier transform

THE OPPORTUNITY

Recursive Asterisms is an AI-assisted mathematical discovery platform in development. It is designed to help people investigate mathematical questions, uncover relationships, develop conjectures, and connect results to the reasoning and evidence behind them. Visual exploration, AI assistance and formal verification would support one shared inquiry. The opportunity is to turn promising ideas into results people can examine, understand and build upon, preserving useful discoveries even when the original question remains open.

THE PROBLEM AND INITIAL CUSTOMER

The need is to carry a promising idea through investigation without losing what supports it. Our own work exposed the effort of reconstructing claims, assumptions and checks across conversations, proof files and notes. [1] The initial customer hypothesis is a small mathematics research group already using AI and formal tools; a group leader could fund adoption. Proposed paid workspaces would connect inquiry, evidence and collaboration. The broader vision includes students and curious explorers. External demand and willingness to pay remain unvalidated.

AN INVARIANT ACROSS REPRESENTATIONS

$$\|Fx\|^2 = \|x\|^2$$

A unitary change of basis preserves the squared norm. [2]

EXAMPLE: A PATTERN, ITS STRUCTURE, ITS PROOF

A mathematical object can become more revealing when its representation changes. Consider eight values of a pattern built from three trigonometric components. The discrete Fourier transform expresses those values in a basis of complex rotations. Each coefficient retains magnitude and phase; the inverse recovers the original values. The reason is structural: the normalized Fourier matrix F has orthonormal columns, so $F^*F = I$. Here * denotes conjugate transpose. This is established mathematics, used to make RA's proposed experience visible. [2]

RA would help a person ask what a representation reveals, develop a conjecture, seek counterexamples, and connect conclusions to evidence. AI would support exploration and explanation; formal tools would check exact statements under declared assumptions. A shared record would distinguish those stages and preserve each result's history. Connections with spectral geometry could then open new mathematical questions. [3, 4]

The counterexample matters too. Discard the Fourier phases and the squared norm survives, but the reconstructed pattern changes. One invariant is not a complete description. The value of exploration includes discovering where a tempting generalization fails.

RECURSIVE ASTERISMS

Explore. Conjecture. Verify.

$$F^*Fx = x$$

F: unitary Fourier matrix · x: eight original values · *: conjugate transpose.
Keep all complex coordinates. Recover the complete finite sequence.

Begin with the pattern



x

Squared norm: 49/9

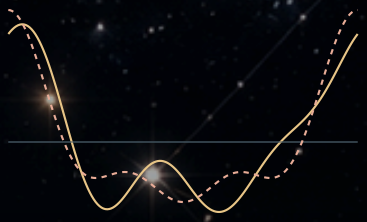
Transform and reconstruct



$F^*X = x$

Every sample is recovered

Discard phase: a counterexample



$F^*|X| \neq x$

Same norm. Different values.

Classical mathematics: retain phase for inversion; preserving a norm alone is insufficient. [2]

ALTERNATIVES AND WHY NOW

AI assistants, proof assistants and research notes already support inquiry. FunSearch demonstrates mathematical discovery; AlphaProof demonstrates formal proof search. [5, 6] RA would connect questions, conjectures, counterexamples, explanations and evidence in one experience. These precedents support the direction; RA's capabilities require validation. A checked proof concerns an exact statement and its assumptions. Novelty needs separate review. [4] Adoption must be earned against existing workflows.

RISKIEST ASSUMPTION

RA can help users reach useful, supported insights with less total effort than existing tools, and they will pay for it. Checking and rework count. Useful discoveries can include counterexamples and intermediate results; original breakthroughs cannot be promised. The current preview illustrates the proposed experience.

WHAT WOULD DISCONFIRM THE OPPORTUNITY?

Interview 12-15 prospective users about recent mathematical investigations. Compare three groups' tasks with their existing workflows. Before testing, agree on useful outcomes and independent review of correctness, evidence and reuse. A provisional target is 25% less total effort at equal or better result quality, counting checking and rework. Weak demand, unreliable claims, review costs exceeding savings, or failure to secure two paid pilot commitments would weaken the thesis. Record novelty separately from proof status. These tests have not yet occurred.

FIRST INVESTMENT

Build a private pilot around one mathematical question, connecting candidate claims, counterexamples, reasoning and reusable results. Bring visual exploration and AI assistance into an inspectable evidence record, with formal checks where appropriate. Test paid individual or team workspaces. Broaden access after evidence supports the value. Preserve authorship and correction history so others can build on each result.

SOURCES

1 Author observations; demand unvalidated
2 MIT: Complex Matrices; Fourier Transform
3 MIT: Eigenvalues and Eigenvectors
4 Lean: Validating a Lean Proof
5 Google DeepMind: FunSearch
6 Google Research: AlphaProof

AI assistance: ChatGPT/Codex assisted with source synthesis, drafting, critique, and visual preparation. Interpretation and final recommendations remain the author's responsibility.

Explore the research
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